# Measurements of the Brightness of the Twilight Sky 

M. J. Koomen, C. Lock, D. M. Packer, R. Scolnik, R. Tousey, and E. O. Hulburt Naval Research Laboratory, Washington, D. C.

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#### Abstract

With a recording photometer of photopic sensitivity, measurements were made of many points in the sky during twilight for solar altitudes $H=+5^{\circ}$ to $-15^{\circ}$ for clear air and no clouds at two stations, one in Maryland, altitude 30 meters, and one on Sacramento Peak, New Mexico, altitude 2800 meters. The sky polarization on the meridian through the sun, and the illumination on a plane at various orientations exposed to the sky, were also recorded. For $H$ from about $-3^{\circ}$ to $-11^{\circ}$ the entire sky changed in brightness at about the same rate of a factor of 10 for each $2^{\circ}$ change in $H$. Except at the horizon the Sacramento Peak sky was about $\frac{2}{3}$ to $\frac{1}{2}$ as bright as the Maryland sky because of clearer air; at the horizon the two were about the same. At Sacramento Peak the ratio of the polarized components reached a minimum of about 0.06 at the zenith for $H=-3^{\circ}$.


IN the present investigation measurements were made of the brightness of various places in the sky during twilight at two stations, one at an altitude of 30 meters and one at an altitude of 2800 meters. Measurements of sky polarization on the meridian through the sun, and of the illumination on a plane at various orientations, during twilight were also included. There appeared to be no previous complete survey of the brightness of the twilight sky. The present work is the latest of several sky brightness investigations that have been carried out by this laboratory. These are, day sky brightness and polarization measurements from the surface ${ }^{1}$ and from an airplane at various altitudes up to 38,000 feet; ${ }^{2,3}$ night sky brightness, but not polarization, measurements from the surface at latitudes $17^{\circ}$ south to $68^{\circ}$ north; ${ }^{4}$ and zenith sky brightness measurements from the surface during a total solar eclipse. ${ }^{5}$ All observations were made as far as possible for a sky free of clouds and an atmosphere fairly free from haze.
The photometer consisted of a nine-stage photomultiplier tube, Type 1P22, Radio Corporation of America, covered with ground glass and a green filter in a $0.6-\mathrm{cm}$ diameter circular aperture, at the focus of a $28 \mathrm{~cm} f: 5$ lens. The field of view was therefore $1.5^{\circ}$ in diameter. The green filter caused the spectral response to be that of the light adapted eye. ${ }^{6}$ The photometer was arranged automatically to sweep the sky in any meridian from horizon to horizon through the zenith in 12 seconds. The photomultiplier was connected to a direct current amplifier and recorder, Brush Development Company, which had adequate speed for following the changes in light intensity during the sweeps. The recorder was kept on scale by manual adjustment of the amplifier. Sensitivity changes to accommodate the

[^0]progress of twilight were obtained by varying the photomultiplier voltage and the size of a circular diaphragm over the lens. The dark current of the photomultiplier was cancelled out by a suitable emf. A polarizing plate could be placed over the lens.

The photometer was calibrated by pointing at a clear daylight sky and comparing its response with the reading of a calibrated blue-filtered Macbeth illuminometer. The relative responses of the photometer for other values of voltage, amplifier gain and lens aperture were obtained by means of the light of a standard tungsten lamp attenuated in a known and nonselective manner. The one-point brightness calibration was therefore sufficient to give a calibration over the entire useful range of the photometer. A small luminous radium phosphor button source was used to insure correctness of calibration during a series of observations.

With this calibration the photometer was used to read, or to give numbers to, the brightness of all places in the twilight sky during all the color changes from daylight to yellow, pinkish, reddish, greenish, etc., and to full night. Since $0.003 \mathrm{ca} \mathrm{ft}^{-2}$ is about the lower limit of the photopic level, the measured brightness values were photometrically correct during the first half of the twilight period. During the second half the values need a correction which probably is not large. The correction was not made, because to make it would require the spectral intensity distribution of the point in the sky under consideration, and such spectral curves are not known exactly for all places in the sky during twilight.

Twilight sky brightness records were made at two stations, one on Sacramento Peak, New Mexico, altitude 2800 meters, and one in the country in Maryland near sea level. The Sacramento Peak measurements were made during seven clear, cloudless, moonless evenings in May and June, 1951, and the Maryland measurements in January, February, and March, 1951. Photometric measurements of sunlight during clear days indicated that the vertical transmission of the atmosphere for sunlight viewed with the light adapted eye was 85 to 90 percent for Sacramento Peak and 75 to 85 percent for Maryland. It may be recalled that the value for one atmosphere of pure Rayleigh air is 90 percent.

Table I. Twilight sky brightness $B$ candles per square foot at Sacramento Peak, New Mexico, altitude 2800 meters.

| H | $\begin{gathered} P=0^{\circ} \\ B \end{gathered}$ | $\stackrel{10^{\circ}}{B}$ | $\begin{gathered} Z=0^{\circ} \\ 30^{\circ} \\ B \end{gathered}$ | $\stackrel{50^{\circ}}{B}$ | $\begin{gathered} 70^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 90^{\circ} \\ B \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+5^{\circ}$ | 1000 | 1000 | 150 | 63 | 35 | 31 |
| + $3^{\circ}$ |  | 560 | 97 | 44 | 24 | 21 |
| $0^{0}{ }^{\circ}$ |  | 150 | 37 | 16.5 | 9.5 | 8.0 |
| - $3^{\circ}$ | ${ }^{20}$ | 15 | 4.7 | 1.9 | 1.3 | 1.0 |
| - $6^{\circ}$ | 0.9 | 0.7 | 0.18 | 0.06 | 0.033 | 0.022 |
| -9 ${ }^{\circ}$ | 0.04 | 0.03 | 0.0256 | 0.0218 | 0.0211 | 0.0375 |
| $-12^{\circ}$ -15 | 0.0218 | 0.0213 | 0.0333 | 0.0314 | 0.0310 | 0.0476 |
| $-15^{\circ}$ | 0.0318 | 0.0310 | 0.0447 | 0.0433 | 0.0425 | 0.0420 |
| $Z=22 \frac{1}{2}^{\circ}$ |  |  |  |  |  |  |
| H | $P=0^{\circ}$ | ${ }_{10}{ }^{\circ}$ | 30 $B$ | ${ }_{50}{ }^{\circ}$ | $70^{\circ}$ $B$ | $\begin{gathered} 90^{\circ} \\ B \end{gathered}$ |
| $+5^{\circ}$ | 600 | 500 | 140 | 60 | 35 | 31 |
| +30 | 350 | 300 | 94 | 41 | 23 | 21 |
| - ${ }^{\circ}$ | 102 | 85 | 34 | 16 | 9.5 | 8 |
| - $3^{\circ}$ | 15 | 14 | 4.6 | 1.9 | 1.3 | 1.0 |
| - $6^{\circ}$ | 0.68 | 0.67 | 0.17 | 0.06 | 0.033 | 0.022 |
| $-9^{\circ}$ | 0.028 | 0.025 | 0.0254 | 0.0218 | 0.0210 | 0.0375 |
| $-12^{\circ}$ | 0.0214 | 0.0210 | 0.0330 | 0.0314 | 0.0310 | 0.0476 |
| $-15^{\circ}$ | 0.0313 | 0.0498 | 0.0446 | 0.0433 | 0.0425 | 0.0420 |
| $Z=45^{\circ}$ |  |  |  |  |  |  |
| H | $P=0^{\circ}{ }^{\circ}$ | $\begin{gathered} 10^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} 30^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} 50^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 70^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 90^{\circ} \\ B \end{gathered}$ |
| + $5^{\circ}$ | 200 | 170 | 100 | 50 | 31 | 31 |
| $+3^{\circ}$ | 145 | 140 | 66 | 35 | 23 | 21 |
| $0^{\circ}$ | 62 | 50 | 23 | 13 | 9.5 | 8.0 |
| - $3^{\circ}$ | 11 | 9.7 | 3.6 | 1.9 | 1.2 | 1.0 |
| $-6^{\circ}$ | 0.4 | 0.35 | 0.10 | 0.046 | 0.031 | 0.022 |
| $-9^{\circ}$ | 0.017 | 0.015 | 0.0234 | 0.0218 | 0.021 | 0.0375 |
| ${ }_{-12^{\circ}}$ | 0.0376 | 0.0370 | 0.032 | 0.0313 | 0.049 | 0.0476 |
| $-15^{\circ}$ | 0.048 | 0.0467 | 0.044 | 0.0432 | 0.0425 | 0.0420 |
| $Z=90^{\circ}$ |  |  |  |  |  |  |
| H | $\mathrm{P}=0^{\text {a }}$ | ${ }_{10}{ }^{\circ}$ | $\stackrel{30^{\circ}}{B}$ | $\begin{gathered} 50^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} 70^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 90^{\circ} \\ B \end{gathered}$ |
| $+5^{\circ}$ | 130 | 100 | 45 | 35 | 30 | 31 |
| $+3^{\circ}$ | 90 | 85 | 38 | 28 | 23 | 21 |
| $0^{\circ}$ | 37 | 34 | 17 | 11 | 9.5 | 8.0 |
| $-3^{\circ}$ | 4.2 | 4.0 | 2.2 | 1.6 | 1.1 | 1.0 |
| $-6^{\circ}$ | 0.11 | 0.08 | 0.05 | 0.04 | 0.025 | 0.022 |
| - $9^{\circ}$ | 0.025 | 0.0246 | 0.022 | 0.0215 | 0.021 | 0.0375 |
| $-12^{\circ}$ | 0.0336 | 0.033 | 0.0315 | 0.0495 | 0.048 | 0.0476 |
| $-15^{\circ}$ | 0.045 | 0.0445 | 0.0435 | 0.0429 | 0.0425 | 0.0420 |
| $Z=135^{\circ}$ |  |  |  |  |  |  |
| H | $P=0^{\circ}$ | ${ }_{10}{ }_{B}^{\circ}$ | $30^{\circ}$ $B$ | ${ }_{50}{ }_{B}^{\circ}$ | ${ }_{70}{ }^{\circ}$ | 90 $B$ |
| + $5^{\circ}$ | 160 | 150 | 72 | 47 | 34 | 31 |
| + $3^{\circ}$ | 100 | 98 | 55 | 35 | 24 | 21 |
| $0^{\circ}$ | 32 | 32 | 20 | 13 | 9.5 | 8.0 |
| $-3^{\circ}$ | 2.4 | 2.1 | 2.1 | 1.5 | 1.1 | 1.0 |
| - $6^{\circ}$ | 0.078 | 0.07 | 0.045 | 0.037 | 0.025 | 0.022 |
| $-9^{\circ}$ | 0.0236 | 0.0235 | 0.022 | 0.0213 | 0.039 | 0.0375 |
| $-12^{\circ}$ | 0.0326 | 0.0323 | 0.0314 | 0.0495 | 0.047 | 0.0476 |
| $-15^{\circ}$ | 0.045 | 0.045 | 0.0436 | 0.0428 | 0.0422 | 0.0420 |
| $Z=180^{\circ}$ |  |  |  |  |  |  |
| H | $P=0^{\circ}$ | $\stackrel{10^{\circ}}{B}$ | $\begin{gathered} 30^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 50_{B}^{\circ} \end{gathered}$ | $\underset{B}{70^{\circ}}$ | $\stackrel{90^{\circ}}{B}$ |
|  | 190 | 170 | 80 | 50 | 35 | 31 |
| $+3^{\circ}$ | 120 | 115 | 60 | 36 | 24 | 21 |
| $0^{\circ}$ | 38 | 37 | 22 | 14 | 9.5 | 8.0 |
| $-3^{\circ}$ | 2.4 | 2.0 | 2.1 | 1.6 | 1.1 | 1.0 |
| $-6^{\circ}$ | 0.078 | 0.068 | 0.043 | 0.035 | 0.025 | 0.022 |
| - $9^{\circ}$ | 0.0236 | 0.023 | 0.0218 | 0.0212 | 0.039 | 0.0375 |
| $-12^{\circ}$ | 0.0328 | 0.033 | 0.0314 | 0.049 | 0.047 | 0.0476 |
| $-15^{\circ}$ | 0.0458 | 0.046 | 0.0442 | 0.043 | 0.0423 | 0.0420 |

The brightness of the place in the sky is denoted by $B$ candles per square foot. $H$ is the altitude of the sun, positive and negative values referring to the sun above and below the horizon, respectively. The position of the place in the sky is designated by the altitude $P$ above the horizon and the bearing $Z$ from the direction of the sun. Other things being equal, it is tacitly assumed that the sky brightness is the same in the two quarterspheres on either side of the meridian through the

Table II. Twilight sky brightness $B$ candles per square foot in Maryland, altitude 30 meters.

| $H$ | $P=0^{\circ}$ | $\underset{B}{10^{\circ}}$ | $\begin{gathered} Z=0^{\circ} \\ 30^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 50^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 70^{\circ} \\ B \end{gathered}$ | $\begin{gathered} 90^{\circ} \\ B \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+5^{\circ}$ | 1000 | 1000 | 350 | 150 | 80 | 43 |
| $+3^{\circ}$ | - | 600 | 220 | 100 | 62 | 35 |
| $0^{\circ}$ | - | 150 | 74 | 35 | 21 | 15 |
| $-3^{\circ}$ | 17 | 15 | 9.5 | 4.0 | 3.0 | 2 |
| $-6^{\circ}$ | 0.8 | 0.7 | 0.3 | 0.12 | 0.08 | 0.06 |
| $-9^{\circ}$ | 0.035 | 0.03 | 0.0272 | 0.024 | 0.0219 | 0.0215 |
| $-12^{\circ}$ | 0.0217 | 0.0215 | 0.036 | 0.0323 | 0.0316 | 0.0312 |
| $-15^{\circ}$ | 0.0328 | 0.0321 | 0.0311 | 0.0462 | 0.0457 | 0.044 |
| $Z=22{\frac{1}{}{ }^{\circ}}^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $\vec{B}$ | $B$ |  | $\boldsymbol{B}$ | B | B |
| $+5^{\circ}$ | 450 | 500 | 240 | 120 | 60 | 43 |
| $+3{ }^{\circ}$ | 290 | 300 | 180 | 78 | 50 | 35 |
| $0^{\circ}$ | 95 | 90 | 64 | 30 | 20 | 15 |
| $-3^{\circ}$ | 13.5 | 13 | 7.7 | 3.7 | 2.5 | 2 |
| $-6^{\circ}$ | 0.6 | 0.65 | 0.3 | 0.1 | 0.075 | 0.06 |
| $-9^{\circ}$ | 0.029 | 0.025 | 0.027 | 0.0233 | 0.0218 | 0.0215 |
| $-12^{\circ}$ | 0.0212 | 0.0211 | 0.035 | 0.0322 | 0.0315 | 0.0312 |
| $-15^{\circ}$ | 0.0319 | 0.0317 | $0.0{ }_{3} 11$ | 0.0462 | 0.0455 | 0.044 |
| $Z=45^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| H | $B$ | B | B | $B$ | $B$ | B |
| $+5^{\circ}$ | 250 | 260 | 160 | 80 | 50 | 43 |
| $+3^{\circ}$ | 170 | 180 | 130 | 60 | 41 | 35 |
| $0^{\circ}$ | 60 | 58 | 50 | 28 | 19 | 15 |
| $-3^{\circ}$ | 9.4 | 10 | 5.8 | 3.4 | 2.2 | 2 |
| $-6^{\circ}$ | 0.42 | 0.35 | 0.20 | 0.08 | 0.062 | 0.06 |
| $-9^{\circ}$ | 0.018 | 0.015 | 0.025 | 0.0228 | 0.0216 | 0.0215 |
| $-12^{\circ}$ | 0.038 | 0.0368 | 0.0337 | 0.0319 | 0.0314 | 0.0312 |
| $-15^{\circ}$ | 0.0312 | 0.0312 | 0.0498 | 0.046 | 0.045 | 0.044 |
| $Z=90^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $B$ | $B$ | B | B | $B$ | B |
| $+5^{\circ}$ | 100 | 94 | 80 | 60 | 45 | 43 |
| $+3^{\circ}$ | 75 | 80 | 66 | 50 | 34 | 35 |
| $0^{\circ}$ | 34 | 34 | 32 | 22 | 17 | 15 |
| $-3^{\circ}$ | 5.5 | 6.0 | 3.5 | 3 | 2 | 2 |
| $-6^{\circ}$ | 0.15 | 0.09 | 0.09 | 0.07 | 0.05 | 0.06 |
| $-9^{\circ}$ | 0.0254 | 0.00245 | 0.0234 | $\mathrm{O}_{0} \mathrm{O}_{2} 2$ | 0.0214 | 0.0215 |
| $-12^{\circ}$ | 0.0335 | 0.0335 | 0.0325 | 0.0316 | 0.0312 | 0.0312 |
| $-15^{\circ}$ | 0.047 | 0.047 | 0.0475 | 0.0452 | 0.0446 | 0.044 |
| $Z=135^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $\boldsymbol{B}$ | $B$ | $B$ | $B$ | $B$ | $B$ |
| $+5^{\circ}$ | 160 | 150 | 100 | 64 | 48 | 43 |
| $+3^{\circ}$ | 110 | 100 | 70 | 50 | 39 | 35 |
| 00 | 39 | 35 | 30 | 20 | 16 | 15 |
| $-3^{\circ}$ | 4.0 | 3.0 | 3.0 | 3.0 | 2 | 2 |
| $-6^{\circ}$ | 0.09 | 0.07 | 0.07 | 0.07 | 0.05 | 0.06 |
| $-9^{\circ}$ | 0.0232 | 0.023 | 0.0225 | 0.022 | 0.0213 | 0.0215 |
| $-12^{\circ}$ | 0.0325 | 0.0325 | 0.032 | 0.0315 | 0.0310 | 0.0312 |
| $-15^{\circ}$ | 0.0456 | 0.0455 | 0.045 | 0.0445 | 0.0443 | 0.044 |
| $Z=180^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $\boldsymbol{H}$ | $\boldsymbol{B}$ | $\boldsymbol{B}$ | B | B | B | $B$ |
|  | 180 | 170 | 110 | 65 | 50 | 43 |
| $+3^{\circ}$ | 120 | 120 | 78 | 54 | 40 | 35 |
| $0^{\circ}$ | 42 | 40 | 35 | 23 | 19 | 15 |
| $-3^{\circ}$ | 2 | 2 | 4 | 3.2 | 2 | 2 |
| $-6^{\circ}$ | 0.09 | 0.08 | 0.07 | 0.07 | 0.05 | 0.06 |
| $-9^{\circ}$ | 0.0232 | 0.023 | 0.0225 | 0.022 | 0.0215 | 0.0215 |
| $-12^{\circ}$ | 0.0325 | 0.0325 | $0_{0.032}$ | 0.0317 | 0.0311 | 0.0312 |
| $-15^{\circ}$ | 0.0456 | 0.0454 | 0.045 | 0.0446 | 0.044 | 0.044 |
| . |  |  |  |  |  |  |

sun and is the same during morning and evening twilight.

From the records over a thousand values of $B$ for each of the two stations were read off and were plotted against $H$ for various values of $P$ and $Z$, smooth curves being drawn through the observed points. From the curves the values of $B$ were taken and are listed in Tables I and II. For illustration, the observed values of $B$ at the zenith at Sacramento Peak are shown in


Fig. 1. Zenith sky brightness values at Sacramento Peak.

Fig. 1, and the smooth curve through them from which the values of Table I were obtained; the points indicate the irregularity due to variations in sky brightness and instrumental inaccuracy.

In Fig. 2 the data of Table II are plotted in smooth curves for six places in the Maryland sky during a period beginning shortly before sunset and extending through twilight. The ordinates are the altitudes of the sun, and the abscissas are the brightness of the place in the sky marked on each curve. On the semi-logarithmic plot of Fig. 2 the curves from about $H=-3^{\circ}$ to $-11^{\circ}$ are approximately straight lines of the same slope. This was also true for the data of Table II for the other places in the sky; likewise for the data of Table I. Therefore, during a certain period of twilight, for altitudes of the sun from about $-3^{\circ}$ to $-11^{\circ}$, the entire sky changed in brightness at approximately the same rate, and at the rate of about a factor of 10 for each $2^{\circ}$ change in $H$. Comparing Tables I and II it is seen that near the horizon the Sacramento Peak sky was of about the same brightness as the Maryland sky, and for places in the sky above $10^{\circ}$ the Sacramento Peak sky was about $\frac{2}{3}$ to $\frac{1}{2}$ the brightness of the Maryland sky, because of the purer air at the higher altitude.
Other facts appear from the data of Tables I and II which may be better seen from contour maps of $B$ over the sky (these maps are not reproduced here). At sunset the darkest part of the sky is in the region of the meridian $Z=90^{\circ}$ and shifts away from the sun as the sun moves below the horizon. Thus, for example, during evening twilight, for equal altitudes the sky in the east is brighter than the sky in the northern and southern direction, all of course being less bright than the sky in the west. This has been well known. The shadow of the
earth in the sky which may be seen rising in the east during the early part of evening twilight when the sky is very clear (and setting in the west during morning twilight) is barely adumbrated in the numerical values of Tables I and II. Te phhenomenon, although easily visible, is relatively faint and probably requires for numerical description more accurate and more closely spaced data than were obtained in the present measurements.

It was the impression of the observers that owing to the clearness of the mountain air the overhead and eastern portions of the sky during evening twilight were much darker relative to the western sky at Sacramento Peak than in Maryland. Although the impression was probably enhanced by color contrasts, the effect is borne out by the brightness values of Tables I and II, which show for example that for equal brightness of the western sky at the two stations in the evening, the zenith sky was about one-half as bright at Sacramento Peak as it was in Maryland.


Fig. 2. Sky brightness values for Maryland.


Fig. 3. Polarization of sky on meridian through sun at Sacramento Peak during evening twilight.

The polarization factor $\rho$ was recorded at Sacramento Peak for the meridian through the sun during twilight. Let $B_{\perp}$ and $B_{11}$ be the two polarized components of $B$ with electric vectors perpendicular and parallel, respectively, to the plane containing the observer, the sun and the point in the sky. Then

$$
\begin{equation*}
B=B_{\perp}+B_{\mathrm{II}} . \tag{1}
\end{equation*}
$$

$\rho$ is defined by

$$
\begin{equation*}
\rho=B_{\mathrm{HI}} / B_{\perp} . \tag{2}
\end{equation*}
$$

Smoothed curves of $\rho$ are given in Fig. 3 for altitudes of the sun $H=+5^{\circ}, 0^{\circ},-3^{\circ},-6^{\circ}$, and $-9^{\circ}$. It is seen in Fig. 3 that after sunset during twilight the region of maximum polarization (i.e., minimum value of $\rho$ ) remained at the zenith within the error of experiment and reached a minimum value $\rho=0.06$ at the zenith for $H=-3^{\circ}$.

At Sacramento Peak, in addition to recording $B$, records of $E$ were made, where $E$ is the illumination in foot candles on a flat surface of opal glass exposed to the sky and oriented in various directions, the orientation being specified by $P$ the angular altitude above the horizon of the normal to the surface and $Z$ the angle between the bearing of the normal and the bearing, or meridian, of the sun. The averaged and smoothed values of $E$ are listed in Table III, in which as before $H$ is the altitude of the sun. Actually $E$ may be obtained by suitable integration of $B$ over the sky for each orientation of the surface; the integration is elementary and tedious. Several of the values of $E$ of Table III were checked* by integration of the values of $B$ of Table I. The check was found to be satisfactory and within the accuracy of the two tables, except in cases where most of the illumination on the opal glass was near grazing incidence. In these cases the observed values of $E$ fell

[^1]Table III. Illumination $E$ foot candles upon a flat surface during twilight at Sacramento Peak, New Mexico, altitude 2800 meters.

| $H$ | $P=0^{\circ}$ | $10^{\circ}$ $E$ | $Z=0^{\circ}$ $30^{\circ}$ $E$ | 50 $E$ | ${ }^{70}{ }^{\circ}$ | ${ }^{90}{ }^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+5^{\circ}$ | 2100 | 2100 | 1900 | 1480 | 900 | 190 |
| $+3^{\circ}$ | 1200 | 1250 | 1200 | 950 | 580 | 125 |
| $0^{\circ}$ | 180 | 180 | 190 | 150 | 108 | 40 |
| $-3^{\circ}$ | 8.8 | 9.8 | 9.8 | 7.6 | 6.4 | 4 |
| $-6^{\circ}$ | 0.5 | 0.52 | 0.52 | 0.42 | 0.27 | 0.125 |
| $-9^{\circ}$ | 0.035 | 0.031 | 0.029 | 0.023 | 0.012 | 0.025 |
| $Z=22 \frac{1}{2}^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | E | $E$ | E | $E$ | $E$ | $E$ |
| $+5^{\circ}$ | 2050 | 2000 | 1800 | 1450 | 840 | 190 |
| $+3^{\circ}$ | 1200 | 1200 | 1100 | 860 | 510 | 125 |
| $0^{\circ}$ | 190 | 180 | 195 | 150 | 98 | 40 |
| $-3^{\circ}$ | 8 | 9.2 | 9.7 | 7.5 | 6.2 | 4 |
| $-6^{\circ}$ | 0.43 | 0.47 | 0.46 | 0.37 | 0.27 | 0.125 |
| $-9^{\circ}$ | 0.025 | 0.025 | 0.024 | 0.021 | 0.011 | 0.025 |
| $Z=45^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $E$ | $E$ | $E$ | $E$ | $E$ | $E$ |
| $+5^{\circ}$ | 1800 | 1450 | 1400 | 1100 | 650 | 190 |
| $+3^{\circ}$ | 950 | 950 | 900 | 670 | 410 | 125 |
| $0^{\circ}$ | 160 | 150 | 140 | 120 | 81 | 40 |
| $-3^{\circ}$ | 6.8 | 6.8 | 7.3 | 6.7 | 5.6 | 4 |
| $-6^{\circ}$ | 0.32 | 0.34 | 0.37 | 0.33 | 0.24 | 0.125 |
| $-9^{\circ}$ | 0.018 | 0.019 | 0.018 | 0.017 | 0.0108 | 0.025 |
| $Z=90^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $E$ | E | E | E | E | E |
| $+5^{\circ}$ | 155 | 150 | 155 | 175 | 180 | 190 |
| $+3^{\circ}$ | 110 | 105 | 110 | 110 | 112 | 125 |
| $0^{\circ}$ | 34 | 37 | 44 | 42 | 44 | 40 |
| $-3^{\circ}$ | 3.5 | 3.4 | 3.7 | 4.2 | 3.9 | 4 |
| $-6^{\circ}$ | 0.14 | 0.15 | 0.17 | 0.155 | 0.15 | 0.125 |
| $-9^{\circ}$ | 0.0264 | 0.0264 | 0.027 | 0.027 | $0.0{ }_{2} 64$ | 0.025 |
| $Z=135^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | E | E | $E$ | E | E | E |
| $+5^{\circ}$ | 110 | 110 | 120 | 130 | 130 | 190 |
| $+3^{\circ}$ | 83 | 77 | 90 | 93 | 93 | 125 |
| $0^{\circ}$ | 26 | 29 | 33 | 35 | 34 | 40 |
| $-3^{\circ}$ | 2 | 1.9 | 2.6 | 3 | 3.4 | 4 |
| $-6^{\circ}$ | 0.05 | 0.064 | 0.064 | 0.09 | 0.087 | 0.125 |
| $-9^{\circ}$ | 0.0221 | 0.0223 | 0.0226 | 0.0226 | 0.0235 | 0.025 |
| $Z=180^{\circ}$ |  |  |  |  |  |  |
|  | $P=0^{\circ}$ | $10^{\circ}$ | $30^{\circ}$ | $50^{\circ}$ | $70^{\circ}$ | $90^{\circ}$ |
| $H$ | $E$ | $E$ | $E$ | $E$ | E | E |
| $+5^{\circ}$ | 112 | 111 | 130 | 130 | 128 | 190 |
| $+3^{\circ}$ | 80 | 84 | 100 | 96 | 96 | 125 |
| $0^{\circ}$ | 25 | 29 | 33 | 35 | 35 | 40 |
| $-3^{\circ}$ | 1.8 | 2.1 | 2.2 | 2.9 | 3.2 | 4 |
| $-6^{\circ}$ | 0.049 | 0.065 | 0.067 | 0.085 | 0.096 | 0.125 |
| $-9^{\circ}$ | 0.0225 | 0.0225 | 0.023 | 0.023 | 0.0236 | 0.025 |

below the calculated values by amounts which reached 50 percent for the maximum discrepancy. The discrepancy was due to the fact that the opal glass was not a true Lambert surface for light near grazing incidence. Therefore, Table III may be considered to have been obtained from Table I by machine computation, the machine being the recording photometer and being a fairly accurate machine except for cases of light near grazing incidence.

In conclusion, it is a pleasure to express our thanks to Mr. Rudolph Cook of the Sacramento Peak Station of Harvard College Observatory for his courteous help to our observing party at Sacramento Peak.


[^0]:    ${ }^{1}$ R. A. Richardson and E. O. Hulburt, Jr., Geophys. Res. 54, 215-227 (1949).
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[^1]:    * The computations were kindly made by Mr. J. H. Hancock of the Mechanics Division of this laboratory.

